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ON THE HYPERBOLA (a+1) x^2- ay^2 = 3a + 3, a >0

M.A. Gopalan[*1], S.Vidhyalakshmi[2], J.Shanthi[3]
Professor, Department of Mathematics, Shrimati Indira Gandhi College,Trichy[*1,2,3]

ABSTRACT

The binary quadratic equation representing hyperbola given (a+1) x2- ay2 = 3a + 3, a >0 is analysed for determining its non-zero distinct integer points. The recurrence relations satisfied by and are given. A few interesting relations among the solutions are presented.

Keywords: Binary quadratic, integer points, hyperbola.

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I. INTRODUCTION

The binary quadratic Diophantine equations offer an unlimited field for research because of their variety [1,2]. For an extensive review of various problems one may refer [3-25]. This communication concerns with yet another interesting binary quadratic equation (a+1) x2- ay2 = 3a + 3, a >0 representing hyperbola for determining its infinitely many non zero integral solutions. Also, a few interesting relations among the solutions are presented .

II. METHOD OF ANALYSIS

The hyperbola under consideration is

$$(a+1)x^2 - ay^2 = 3a+3,\ a > 0 \tag{1}$$

Introducing the linear transformation

$$x = X + aT, \quad and \quad y = X + (a+1)T \tag{2}$$

In (1), it is written as

$$X^2 = (a^2 + a)T^2 + 1$$

Which is a Pellian equation, whose general solution $(\breve{X}_n , \breve{T}_n)$ is given by

$$\breve{X}_n = \frac{1}{2} f_n$$

$$\breve{T}_n = \frac{1}{2\sqrt{a^2+a}} g_n$$

Where $f_n = \left\{\left[(2a+1) + \left(\sqrt{a^2+a}\right)2\right]^{n+1} + \left[(2a+1) - \left(\sqrt{a^2+a}\right)2\right]^{n+1}\right\}$

$$g_n = \left\{\left[(2a+1) + \left(\sqrt{a^2+a}\right)2\right]^{n+1} - \left[(2a+1) - \left(\sqrt{a^2+a}\right)2\right]^{n+1}\right\}$$

In view of (2), the integer values of x and y satisfying (1) are given by

$$x_{n+1} = (a+1)f_n + \frac{(2a^2+3a)}{2\sqrt{a^2+a}} g_n \tag{3}$$

$$y_{n+1} = \frac{2a+3}{2} f_n + \frac{(2a^2+4a+2)}{2\sqrt{a^2+a}} g_n \tag{4}$$

The recurrence relations satisfied by the x and y values of (1) are given by

$$x_{n+3} - (4a+2)x_{n+2} + x_{n+1} = 0$$

With $x_0 = 2a+2 \quad \& \quad x_1 = 8a^2 + 12a + 2$

Similarly,

$$y_{n+3} - (4a+2)y_{n+2} + y_{n+1} = 0$$

With $y_0 = 2a+3$ & $y_1 = 8a^2+16a+7$

A few interesting relations among the solutions are as follows:

(1). $x_{n+2} = (2a+1)x_{n+1} + 2a\, y_{n+1}$

(2). $x_{n+3} = (8a^2+8a+1)x_{n+1} + (8a^2+4a)\, y_{n+1}$

(3). $y_{n+2} = (2a+2)x_{n+1} + (2a+1)\, y_{n+1}$

(4). $y_{n+3} = (8a^2+12a+4)x_{n+1} + (8a^2+8a+1)\, y_{n+1}$

(5). $x_{n+3} = x_{n+1} + 4a\, y_{n+2}$

(6). $y_{n+2} = x_{n+1} + y_{n+1} + x_{n+2}$

(7). $x_{n+1}\, y_{n+2} + 2ay_{n+1}^2 = 2(a+1)x_{n+1}^2 + x_{n+2}\, y_{n+1}$

(8). $\frac{4(a+1)^3 x_{2n+2} - a(4a^2+10a+6)y_{2n+2} + (6a^2+14a+8)}{(3a^2+7a+4)}$ is a Perfect square.

(9). $6\left[\frac{4(a+1)^3 x_{2n+2} - a(4a^2+10a+6)y_{2n+2} + (6a^2+14a+8)}{(3a^2+7a+4)}\right]$ is a Nasty number.

(10). $\frac{4(a+1)^3 x_{3n+3} - a(2a+2)(2a+3)y_{3n+3} + 3f_n(3a^2+7a+4)}{(3a^2+7a+4)} = f_n^3$ is cubical number.

III. REMARKABLE OBSERVATION

Let $\alpha_{n+1} = (4a+6)x_{n+1} - (4a+4)y_{n+1}$

$$\beta_{n+1} = 4(a+1)^3 x_{n+1} - (2a+3)(2a^2+2a)y_{n+1}$$

Note that the pair $(\alpha_{n+1}, \beta_{n+1})$ satisfies the hyperbola

$$((3a+4)\beta_{n+1})^2 = (a^2+a)[(3a^2+7a+4)\alpha_{n+1}]^2 + (6a+8)^2(3a^2+7a+4)^2$$

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